**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: The probability that the service manager cannot meet his commitment is the probability that the time required for servicing the transmission is greater than 50 minutes. We can find this probability using the normal CDF.

P(X > 50) = 1 - P(X <= 50) = 1 - Φ((50-45)/8) = 1 - Φ(0.625) = 1 - 0.734 = 0.266

Therefore, the probability that the service manager cannot meet his commitment is 0.266, So the ans is B.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: A) Probability of employees > 44 = Pr(x>44)=1-Pr

Z=(X-µ)/σ = (x-38)/6

Pr(x<=44) = Pr(z<=44-38)/6)=Pr(Z<=1)=0.84134 = 84.134%

Probability that employees will be greater than 44 = 100-84.134 = 15.866

Probability of employees between 38 and 44 = Pr(x<=44) – Pr(x>=38)

Here , Pr(x<=44) = 0.84134

Pr(x>=38)=Pr(z>=(38-38)/6)=Pr(z>=0)=0.5

Therefore, Pr(x<=44) – Pr(x>=38) = 0.84134 – 0.5 = 0.34134 = 34.134%

So the statement ‘ More employees at the processing centre are older than 44 than between 38 and 44’ is True.

1. Probability of employees less than 30 = Pr(X<30)

Z=(X- μ)/ σ = (30-38)/6

Pr(x<30) = Pr(z<(30-38)/6) = Pr(z<-1.3333) = 0.09176 = 9.17%

So the number of employees with probability 0.0917 of them being under 30 = 400\*0.0917 = 36.68 = 36

The statement ‘ Training program for employees under the age of 30 at the center would be expected to attract about 36 employees’ is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: We know that if X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are two independent Random variables than X+Y~N(μ1 + μ2, σ12 + σ22) and X-Y ~ N(μ1 + μ2, σ12 + σ22)

Similarly if Z=ax+by , Where X and Y are defined above i.e z is linear combination of X and Y then

Z~ N(aμ1 + bμ2 aσ12 +b σ22)

Therefore in this question

2X1~N(2 σ,4 σ) and X1+X2 ~ N(μ+ μ , σ2) ~ N(2 μ , 2 σ2)

2X1 – (X1+X2) = N(4 μ,6 σ2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: Z\*σ + µ = X

Z(-0.005)\*20+100 = (2.57)\*20+100 = 151.4

Z(0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So the answer is option D

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Mean profit is Rs. 540 Million Std deviaton is Rs. 225 Million

Range is RS.

State norms interval (0.95, 540, 225)

Range is RS 99.0081034 , 980.991896

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: Formula X=X=μ + Zσ; where in from z table, 5 percentile = -1.645

X= 540(-1.645)\*225

X= 169.875

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Probability od division 1 making a loss p(X<0) Stats norm. cdf (0, 5, 3)

0.0477903

probability od division 2 making a loss p(X<0) Stats.norm. cdf(0, 7, 4)

0.0400591